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Vacuum Polarisation Tensors in Constant Electromagnetic Fields: Part II

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Abstract

In the second part of this series we apply the "string-inspired" technique to the calculation of one-loop amplitudes involving both vectors and axialvectors, as well as a general constant electromagnetic background field. The vector-axialvector two-point function in a constant field is calculated explicitly.

1 Introduction: Standard Model Processes in Constant Electromagnetic Fields

In the first part of this series [1] we explained in detail how the "string-inspired" technique [2, 3, 4, 5] can be used for the efficient calculation of one-loop scalar/spinor QED N - photon amplitudes in a constant external field [6, 7, 8, 9, 10, 11, 12, 13, 14]. As an application we calculated the scalar and spinor QED vacuum polarisation tensors in a constant field.

In the present sequel, we extend this analysis to the case of mixed vector – axialvector amplitudes in a constant electromagnetic field. Amplitudes of this type have, in recent years, been much investigated in connection with photon-neutrino processes. In the standard model, photon-neutrino interactions appear at the one-loop level. In vacuum, a typical example would be the diagram shown in fig. 1, contributing to the process $\gamma\gamma \to \nu\bar{\nu}$.

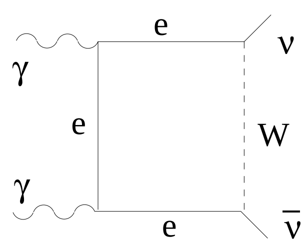

Figure 1: Diagram contributing to $\gamma\gamma \to \nu\bar{\nu}$.

The $2 \to 2$ processes $\gamma\gamma \to \nu\bar{\nu}$, $\gamma\nu \to \gamma\nu$ and $\nu\bar{\nu} \to \gamma\gamma$ were considered already before the advent of the standard model using the Fermi theory [15]. However in the Fermi limit they vanish due to the Landau-Yang theorem, as was noted by Gell-Mann [16] (for massless neutrinos, and with both photons on-shell). In the standard model this suppression manifests itself by factors of $\frac{\omega}{M_W}$, where ω is the center-of-mass energy and M_W the W boson mass.

There is no such suppression for processes involving two neutrinos and more than two photons, which in vacuum are therefore more important at low energies than the four-leg processes [17, 18, 19, 20, 21]. Many more photon-neutrino processes become possible if one admits neutrino masses or anomalous magnetic dipole moments [22].

In astrophysical environments it is often not realistic to consider these processes as occuring in vacuum. Plasma effects must be taken into account, as well as the presence of magnetic fields which, at the surface of neutron stars, have recently been found to surpass the "critical" magnetic field strength $B_{\rm crit} = \frac{m_e^2}{e} = 4.41 \times 10^{13}$ Gauss [23]. Of particular interest are then processes which do not occur in vacuum but become possible in a medium or B-field. An important example is the plasmon decay $\gamma \to \nu\bar{\nu}$ [24, 25], believed to be an important source for neutrino production in some types of stars [26]. Similarly the Cherenkov process $\nu \to \nu\gamma$ becomes possible, although it turns out to be of lesser astrophysical relevance [27, 22]. For processes of this type the magnetic field plays a double role. Firstly, it provides an effective photon-neutrino coupling via intermediate charged particles [27, 28, 29]. Secondly, by modifying the photon dispersion relations it opens up phase space for neutrino-photon reactions of the type $1 \to 2+3$.

Similarly one would expect the magnetic field to remove the Fermi limit suppression of the above $2 \to 2$ processes. This has recently been verified both for the weak [30, 31] and strong field cases [32].

In the standard model the effective coupling is provided by the diagrams shown in fig. 2(a) and 2(b). The double line represents the electron propagator in the presence of the B-field [1] .

In the limit of infinite gauge-boson masses both diagrams can be replaced by diagram fig. 2(c). The amplitude then effectively reduces to a photonic amplitude with one of the photons replaced by the neutrino current.

One is thus led to the study of vector – axialvector amplitudes in a constant field. In the string-inspired formalism, various different representations have been derived for axialvector couplings [33, 34, 35]. We will use here the one proposed in [35] and elaborated in [36], which has the advantage of avoiding the separation of these amplitudes into their real and imaginary parts.

Extending that work to the case where an additional constant electromagnetic background field is present will enable us to perform the first calculation of the vector – axialvector amplitude in a general such field. This amplitude is of relevance for various of the above processes. It has been obtained previously for the magnetic [29, 22, 37] and crossed field [27, 37] special cases.

The organization of the paper is the following. In the second chapter, we shortly review the worldline representation of vector - axialvector amplitudes proposed in [35, 36]. We then extend this formalism to the inclusion of

[1] I thank A.N. Ioannisian for providing this figure.

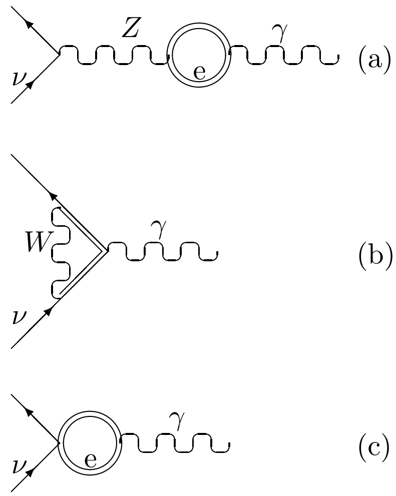

Figure 2: Neutrino-photon coupling in an external magnetic field. (a) Z-A-mixing. (b) Penguin diagram (only for ν_e). (c) Effective coupling in the Fermi limit.

constant external fields in chapter three. Chapter four contains a detailed calculation of the vector - axialvector amplitude in a constant field. The result is discussed in chapter five, as well as possible generalizations.

2 Worldline Representation of the Vector-Axialvector Effective Action

In [35] the following first-quantized path integral representation was derived for the one-loop effective action induced by a Dirac fermion loop for a background vector field A and axial vector field A_5 (both abelian) [2],

[2] We work initially in the Euclidean with a positive definite metric $g_{\mu\nu} = \mathrm{diag}(++++)$. The Euclidean field strength tensor is defined by $F^{ij} = \varepsilon_{ijk} B_k, i, j = 1, 2, 3$, $F^{4i} = -iE_i$, its dual by $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F^{\alpha\beta}$ with $\varepsilon^{1234} = 1$. To obtain the corresponding Minkowski space amplitudes replace $g_{\mu\nu} \to \eta_{\mu\nu} = \mathrm{diag}(-+++)$, $k^4 \to -ik^0, T \to is, \varepsilon^{1234} \to i\varepsilon^{1230}, \varepsilon^{0123} = 1, F^{4i} \to F^{0i} = E_i, \tilde{F}^{\mu\nu} \to -i\tilde{F}^{\mu\nu}$.

$$
\begin{aligned}
\Gamma[A,A_5] &= \ln \mathrm{Det}[\not p + e\not A + e_5\gamma_5 \not A_5 - im] \\
&= -\frac{1}{2}\int_0^\infty \frac{dT}{T}\,\mathrm{e}^{-m^2T}\int \mathcal{D}x \int \mathcal{D}\psi\ \mathrm{e}^{-\int_0^T d\tau\, L(\tau)} \qquad (2.1)\\
L &= \frac{1}{4}\dot x^2 + \frac{1}{2}\psi\cdot\dot\psi + ie\dot x\cdot A - ie\psi\cdot F\cdot\psi \\
&\quad + ie_5\hat\gamma_5\Bigl(-2\dot x\cdot\psi\psi\cdot A_5 + \partial\cdot A_5\Bigr) + (D-2)e_5^2A_5^2
\end{aligned}
\tag{2.2}
$$

Here T denotes the usual Schwinger proper-time for the loop fermion, $\int \mathcal{D}x$ the integral over the space of all closed loops in spacetime with periodicity T, and $\int \mathcal{D}\psi$ a Grassmann path integral representing the spin of the loop fermion. The boundary conditions on the Grassmann path integral are, after expansion of the interaction exponential, determined by the power of $\hat\gamma_5$ appearing in a given term; they are periodic (antiperiodic) with period T if that power is odd (even). After the boundary conditions are determined $\hat\gamma_5$ can be replaced by unity.

Note that, although we will consider only the four-dimensional case in the present work, the Lagrangian has an explicit dependence on the spacetime dimension D through dimensional regularisation [36].

Before evaluating this double path integral one must eliminate the zero mode(s) contained in it. For the N - photon amplitude, the Grassmann path integral has to be taken with antiperiodic boundary conditions, so that only $\int \mathcal{D}x$ has a zero mode. This zero mode we eliminated by fixing the average position $x_0^\mu \equiv \frac{1}{T}\int_0^T d\tau\, x^\mu(\tau)$ of the loop,

$$
\begin{aligned}
x^\mu(\tau) &= x_0^\mu + y^\mu(\tau) \\
\int \mathcal{D}x &= \int dx_0 \int \mathcal{D}y
\end{aligned}
\tag{2.3}
$$

When appearing with periodic boundary conditions, the Grassmann path integral has a zero mode as well. This can be removed analogously,

$$
\begin{aligned}
\psi^\mu(\tau) &= \psi_0^\mu + \xi^\mu(\tau) \\
\int_0^T d\tau\, \xi^\mu(\tau) &= 0
\end{aligned}
\tag{2.4}
$$

The zero mode integration then produces the ε - tensor expected for a fermion loop with an odd number of axial insertions,

$$\int d^4\psi_0 \psi_0^\mu \psi_0^\nu \psi_0^\kappa \psi_0^\lambda = \varepsilon^{\mu\nu\kappa\lambda} \tag{2.5}$$

The reduced path integrals will, following the recipes of the "string-inspired" formalism, be evaluated using Green's functions in τ - space. For the case of an even number of axial vectors the Wick contraction rules are the same as in the QED case treated in part I,

$$\langle y^\mu(\tau_1)\, y^\nu(\tau_2)\rangle = -g^{\mu\nu} G_B(\tau_1,\tau_2) \tag{2.6}$$

$$\langle \psi^\mu(\tau_1)\, \psi^\nu(\tau_2)\rangle = \frac{1}{2} g^{\mu\nu} G_F(\tau_1,\tau_2) \tag{2.7}$$

where

$$\begin{aligned} G_B(\tau_1,\tau_2) &= \mid \tau_1 - \tau_2 \mid - \frac{(\tau_1-\tau_2)^2}{T} \\ G_F(\tau_1,\tau_2) &= \mathrm{sign}(\tau_1-\tau_2) \end{aligned} \tag{2.8}$$

In the case of an odd number of axialvectors the Grassmann Wick contraction rule (2.7) has to be replaced by

$$\langle \xi^\mu(\tau_1)\, \xi^\nu(\tau_2)\rangle = \frac{1}{2} g^{\mu\nu} \dot{G}_B(\tau_1,\tau_2) \tag{2.9}$$

(A "dot" always refers to a derivative in the first variable.) The free Gaussian path integral determinants are, in our conventions,

$$\int \mathcal{D}y\, \mathrm{e}^{-\int_0^T d\tau\, \frac{1}{4}\dot{y}^2} = (4\pi T)^{-\frac{D}{2}} \tag{2.10}$$

$$\int_A \mathcal{D}\psi\, \mathrm{e}^{-\int_0^T d\tau\, \frac{1}{2}\psi\cdot\dot{\psi}} = 4 \tag{2.11}$$

$$\int_P \mathcal{D}\xi\, \mathrm{e}^{-\int_0^T d\tau\, \frac{1}{2}\xi\cdot\dot{\xi}} = 1 \tag{2.12}$$

In writing them we have specialized to the four-dimensional case, leaving a D - dependence only for the free y - path integral (2.10). This anticipates the dimensional regularization. See [36] for the case of an arbitrary (even) spacetime dimension.

3 One-Loop Vector-Axialvector Amplitudes in a Constant Field

In the same way as it was done in part I for the pure vector case, we can obtain from (2.2) the M vector $-$ N axialvector amplitude by specializing to backgrounds consisting of sums of plane waves with definite polarizations,

$$
\begin{aligned}
A_\mu(x) &= \sum_{i=1}^{M} \varepsilon_{i\mu} \mathrm{e}^{ik_i\cdot x} \\
A_{5\mu}(x) &= \sum_{i=1}^{N} \varepsilon_{5i\mu} \mathrm{e}^{ik_{5i}\cdot x}
\end{aligned}
\tag{3.1}
$$

and picking out the term containing every $\varepsilon_i, \varepsilon_{5i}$ once. The only slight complication compared to the QED case is due to the term quadratic in A_5 appearing in the worldline Lagrangian (2.2). To be able to define an analogue of the photon vertex operator

$$
V_A^{\frac{1}{2}}[k,\varepsilon] = \int_0^T d\tau \Bigl(\varepsilon \cdot \dot{x} + 2i\varepsilon \cdot \psi k \cdot \psi\Bigr)\, \mathrm{e}^{ikx} \tag{3.2}
$$

for the axial coupling, it is convenient to linearize this term through the introduction of an auxiliary path integration [36]:

$$
\begin{aligned}
\exp\Bigl[-(D-2)e_5^2 \int_0^T d\tau A_5^2\Bigr] &= \int \mathcal{D}z \\
&\quad \times \exp\Bigl[-\int_0^T d\tau \Bigl(\frac{z^2}{4} + ie_5\sqrt{D-2}z \cdot A_5\Bigr)\Bigr]
\end{aligned}
\tag{3.3}
$$

The Wick contraction rule for this auxiliary field is simply

$$
\langle z^\mu(\tau_1) z^\nu(\tau_2)\rangle = 2g^{\mu\nu}\delta(\tau_1-\tau_2) \tag{3.4}
$$

This allows us to define an axial-vector vertex operator as follows,

$$
V_{A_5}[k,\varepsilon] \equiv \hat{\gamma}_5 \int_0^T d\tau \Bigl(i\varepsilon \cdot k + 2\varepsilon \cdot \psi \dot{x} \cdot \psi + \sqrt{D-2}\,\varepsilon \cdot z\Bigr)\, \mathrm{e}^{ik\cdot x} \tag{3.5}
$$

We can then write the one-loop M vector – N axialvector amplitude in terms of Wick contractions of vertex operators:

$$
\begin{aligned}
\Gamma[\{k_i,\varepsilon_i\},\{k_{5j},\varepsilon_{5j}\}] &= -\frac{1}{2}N_{A,P}(-i)^{M+N}e^M e_5^N \\
&\times\int_0^\infty \frac{dT}{T}\, e^{-m^2T}(4\pi T)^{-\frac{D}{2}}\Big\langle V_A^{\frac{1}{2}}[k_1,\varepsilon_1]\dots \\
&\dots V_A^{\frac{1}{2}}[k_M,\varepsilon_M]V_{A_5}[k_{51},\varepsilon_{51}]\dots V_{A_5}[k_{5N},\varepsilon_{5N}]\Big\rangle_{A,P}
\end{aligned}
\tag{3.6}
$$

The Wick contractions are done using (2.6), (3.4), and either (2.7) or (2.9), depending on the boundary conditions. This can still be done in closed form [36], however the result is lengthy.

Let us now introduce an additional background vector field $\bar{A}^\mu(x)$ with constant field strength tensor $\bar{F}_{\mu\nu}$. In Fock–Schwinger gauge centered at x_0 [6] its contribution to the worldline Lagrangian (2.2) can be written as $\Delta L = \frac{1}{2}\, ie\, y^\mu \bar{F}_{\mu\nu}\dot{y}^\nu - ie\,\psi^\mu\bar{F}_{\mu\nu}\psi^\nu$. It can therefore be absorbed into the kinetic part of Lagrangian, and be taken into account by a change of the free worldline propagators. This leads to a replacement of $G_B, \dot{G}_B, G_F$ by ([8, 10, 11]; see also [7])

$$
\begin{aligned}
\mathcal{G}_B(\tau_1,\tau_2) &= \frac{T}{2\mathcal{Z}^2}\biggl(\frac{\mathcal{Z}}{\sin(\mathcal{Z})}\,\mathrm{e}^{-i\mathcal{Z}\dot{G}_{B12}}+i\mathcal{Z}\dot{G}_{B12}-1\biggr)\\
\dot{\mathcal{G}}_B(\tau_1,\tau_2) &= \frac{i}{\mathcal{Z}}\biggl(\frac{\mathcal{Z}}{\sin(\mathcal{Z})}\,\mathrm{e}^{-i\mathcal{Z}\dot{G}_{B12}}-1\biggr)\\
\mathcal{G}_F(\tau_1,\tau_2) &= G_{F12}\frac{\mathrm{e}^{-i\mathcal{Z}\dot{G}_{B12}}}{\cos(\mathcal{Z})}
\end{aligned}
\tag{3.7}
$$

where we have defined $\mathcal{Z}\equiv eFT$ (omitting the "bar"). The presence of the background field thus modifies the Wick contraction rules eqs.(2.6),(2.7),(2.9) to

$$\langle y^\mu(\tau_1)y^\nu(\tau_2)\rangle = -\mathcal{G}_B^{\mu\nu}(\tau_1,\tau_2) \tag{3.8}$$

$$\langle \psi^\mu(\tau_1)\psi^\nu(\tau_2)\rangle = \frac{1}{2}\mathcal{G}_F^{\mu\nu}(\tau_1,\tau_2) \tag{3.9}$$

$$\langle \xi^\mu(\tau_1)\xi^\nu(\tau_2)\rangle = \frac{1}{2}\dot{\mathcal{G}}_B^{\mu\nu}(\tau_1,\tau_2) \tag{3.10}$$

The free Gaussian path integral determinants (2.10),(2.11),(2.12) also become field dependent [6, 11]:

$$\int \mathcal{D}y \, \mathrm{e}^{-\int_0^T d\tau \left(\frac{1}{4}\dot{y}^2 + \frac{1}{2}ie\, y^\mu F_{\mu\nu}\dot{y}^\nu\right)} = (4\pi T)^{-\frac{D}{2}} \det{}^{-\frac{1}{2}}\left[\frac{\sin(\mathcal{Z})}{\mathcal{Z}}\right] \quad (3.11)$$

$$\int_A \mathcal{D}\psi \, \mathrm{e}^{-\int_0^T d\tau \left(\frac{1}{2}\psi\cdot\dot{\psi} - ie\, \psi^\mu F_{\mu\nu}\psi^\nu\right)} = 4 \det{}^{\frac{1}{2}}\left[\cos\mathcal{Z}\right] \quad (3.12)$$

$$\int_P \mathcal{D}\xi \, \mathrm{e}^{-\int_0^T d\tau \left(\frac{1}{2}\xi\cdot\dot{\xi} - ie\, \xi^\mu F_{\mu\nu}\xi^\nu\right)} = \det{}^{\frac{1}{2}}\left[\frac{\sin(\mathcal{Z})}{\mathcal{Z}}\right] \quad (3.13)$$

Note, however, that for the case of periodic Grassmann boundary conditions this field dependence cancels out between the coordinate and Grassmann path integrals. This cancellation can be understood as a consequence of the fact that the two path integrals are related by worldline supersymmetry [2, 4, 36]. It does not occur in the antiperiodic case since here the supersymmetry is broken by the boundary conditions.

The above trigonometric expressions should be understood as power series in the field strength tensor. In section 3.2 of part I we gave explicit expressions for the generalized worldline Green's functions and determinants in terms of $\mathbb{1}, F, \tilde{F}, F^2$, and the two standard Maxwell invariants $f = \frac{1}{4}F\cdot F, g = \frac{1}{4}F\cdot\tilde{F}$.

This is all, then, which we have to know to calculate one-loop processes involving any numbers of (abelian) vectors, axialvectors as well as a constant external (vector) field. While in the present paper we will consider only the amplitude case, the formalism can be applied as well to the calculation of the effective action itself in the inverse higher derivative expansion (see [6, 38, 8, 9, 13, 35]).

4 Worldline Calculation of the Vector – Axialvector Amplitude in a Constant Field

As an explicit example, we calculate the vector – axialvector two-point function in a constant field. According to the above we can represent this amplitude as follows,

$$
\begin{aligned}
\langle A_\mu(k_1)A_{5\nu}(k_2)\rangle &= \frac{1}{2}\int_0^\infty \frac{dT}{T}\, e^{-m^2T}\int\mathcal{D}x\int\mathcal{D}\psi \\
&\times \exp\Bigl\{-\int_0^T d\tau\Bigl[\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}+\frac{i}{2}e\,x\cdot F\cdot\dot{x}-ie\,\psi\cdot F\cdot\psi\Bigr]\Bigr\} \\
&\times \int_0^T d\tau_1\Bigl(\dot{x}_\mu(\tau_1)+2i\psi_\mu(\tau_1)k_1\cdot\psi(\tau_1)\Bigr)\mathrm{e}^{ik_1\cdot x_1} \\
&\times \int_0^T d\tau_2\Bigl(ik_{2\nu}+2\psi_\nu(\tau_2)\dot{x}(\tau_2)\cdot\psi(\tau_2)\Bigr)\mathrm{e}^{ik_2\cdot x_2}
\end{aligned}
\tag{4.1}
$$

Note that this expression is already manifestly gauge invariant, i.e. transversal in the vector index. If one multiplies the right hand side by k_1^μ then the integrand of the vector vertex operator becomes a total derivative in τ_1, so that the integral vanishes by periodicity. This mechanism is well-known from string theory. Nothing analogous holds for the axialvector vertex operator.

This amplitude is finite, so that we can set $D=4$ in its evaluation. As a first step, the zero-modes of both path integrals are separated out according to eqs.(2.3),(2.4), and the Grassmann zero mode integrated out using eq.(2.5). All terms which do not contain all four zero mode components precisely once give zero. To explicitly perform this integration we note that by eq.(2.4) we can rewrite, in the exponent of eq.(4.1),

$$
\int_0^T d\tau\,\psi(\tau)\cdot F\cdot\psi(\tau) = T\psi_0\cdot F\cdot\psi_0+\int_0^T d\tau\,\xi(\tau)\cdot F\cdot\xi(\tau) \tag{4.2}
$$

Thus for the case at hand the Grassmann zero mode integral can appear in the following three forms,

$$
\begin{aligned}
\int d^4\psi_0\,\mathrm{e}^{ieT\psi_0\cdot F\cdot\psi_0} &= -\frac{(eT)^2}{2}\varepsilon_{\mu\nu\kappa\lambda}F_{\mu\nu}F_{\kappa\lambda} = -(eT)^2F\cdot\tilde{F} \\
\int d^4\psi_0\,\mathrm{e}^{ieT\psi_0\cdot F\cdot\psi_0}\psi_{0\mu}\psi_{0\nu} &= ieT\varepsilon_{\mu\nu\kappa\lambda}F_{\kappa\lambda} = 2ieT\tilde{F}_{\mu\nu} \\
\int d^4\psi_0\,\mathrm{e}^{ieT\psi_0\cdot F\cdot\psi_0}\psi_{0\mu}\psi_{0\nu}\psi_{0\kappa}\psi_{0\lambda} &= \varepsilon_{\mu\nu\kappa\lambda}
\end{aligned}
\tag{4.3}
$$

In the next step, both path integrations are performed using the field-dependent Wick contraction rules eqs. (3.8) and (3.10). This results in the following parameter integral representation for the vector – axialvector vacuum polarisation tensor [3]

$$
\begin{aligned}
\Pi_5^{\mu\nu}(k) &= \frac{ee_5}{8\pi^2}\int_0^\infty \frac{dT}{T^3}\,\mathrm{e}^{-m^2T}\int_0^T d\tau_1 d\tau_2\; J_5^{\mu\nu}(\tau_1,\tau_2)\mathrm{e}^{-k\cdot\bar{\mathcal{G}}_{12}\cdot k} \\
J_5^{\mu\nu}(\tau_1,\tau_2) &= \Bigl[\ddot{\mathcal{G}}_{12}^{\mu\alpha} - (\dot{\mathcal{G}}_{21}^{\alpha\beta} - \dot{\mathcal{G}}_{22}^{\alpha\beta})(\dot{\mathcal{G}}_{11}^{\mu\rho} - \dot{\mathcal{G}}_{12}^{\mu\rho})k_\beta k_\rho\Bigr]\Bigl(i\tilde{\mathcal{Z}}_{\nu\alpha} - \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\dot{\mathcal{G}}_{22}^{\nu\alpha}\Bigr) \\
&+\frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}(\dot{\mathcal{G}}_{11}^{\mu\rho} - \dot{\mathcal{G}}_{12}^{\mu\rho})k_\rho k_\nu + k_\nu k_\rho\Bigl(i\tilde{\mathcal{Z}}_{\mu\rho} - \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\dot{\mathcal{G}}_{11}^{\mu\rho}\Bigr) + k_\rho k_\sigma(\dot{\mathcal{G}}_{21}^{\alpha\rho} - \dot{\mathcal{G}}_{22}^{\alpha\rho}) \\
&\times\Bigl[\varepsilon_{\mu\sigma\nu\alpha} + i(\dot{\mathcal{G}}_{22}^{\nu\alpha}\tilde{\mathcal{Z}}_{\mu\sigma} - \dot{\mathcal{G}}_{12}^{\sigma\alpha}\tilde{\mathcal{Z}}_{\mu\nu} + \dot{\mathcal{G}}_{12}^{\sigma\nu}\tilde{\mathcal{Z}}_{\mu\alpha} + \dot{\mathcal{G}}_{12}^{\mu\alpha}\tilde{\mathcal{Z}}_{\sigma\nu} - \dot{\mathcal{G}}_{12}^{\mu\nu}\tilde{\mathcal{Z}}_{\sigma\alpha} + \dot{\mathcal{G}}_{11}^{\mu\sigma}\tilde{\mathcal{Z}}_{\nu\alpha}) \\
&-\frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}(\dot{\mathcal{G}}_{11}^{\mu\sigma}\dot{\mathcal{G}}_{22}^{\nu\alpha} - \dot{\mathcal{G}}_{12}^{\mu\nu}\dot{\mathcal{G}}_{12}^{\sigma\alpha} + \dot{\mathcal{G}}_{12}^{\mu\alpha}\dot{\mathcal{G}}_{12}^{\sigma\nu})\Bigr]
\end{aligned}
\tag{4.4}
$$

where $k = k_1 = -k_2, \tilde{\mathcal{Z}} \equiv eT\tilde{F}, \bar{\mathcal{G}}_{12} \equiv \mathcal{G}(\tau_1,\tau_2) - \mathcal{G}(\tau,\tau)$ (see part I). As in the vector – vector case, it is useful to perform a partial integration on the one term involving $\ddot{\mathcal{G}}_{12}$, leading to the replacement

$$\ddot{\mathcal{G}}_{12}^{\mu\alpha} \rightarrow \dot{\mathcal{G}}_{12}^{\mu\alpha}k\cdot\dot{\mathcal{G}}_{12}\cdot k \tag{4.5}$$

By this partial integration, and the removal of some terms which cancel against each other, $J_5^{\mu\nu}(\tau_1,\tau_2)$ gets replaced by

$$
\begin{aligned}
&k^\rho k^\sigma\Bigl[\dot{\mathcal{G}}_{12}^{\mu\alpha}\dot{\mathcal{G}}_{12}^{\rho\sigma} + (\dot{\mathcal{G}}_{21}^{\alpha\sigma} - \dot{\mathcal{G}}_{22}^{\alpha\sigma})\dot{\mathcal{G}}_{12}^{\mu\rho}\Bigr]\Bigl(i\tilde{\mathcal{Z}}^{\nu\alpha} - \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\dot{\mathcal{G}}_{22}^{\nu\alpha}\Bigr) - \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\dot{\mathcal{G}}_{12}^{\mu\rho}k^\rho k^\nu \\
&+ik^\nu k^\rho\tilde{\mathcal{Z}}^{\mu\rho} + k^\rho k^\sigma(\dot{\mathcal{G}}_{21}^{\alpha\rho} - \dot{\mathcal{G}}_{22}^{\alpha\rho})\Bigl[\varepsilon^{\mu\sigma\nu\alpha} + \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}(\dot{\mathcal{G}}_{12}^{\mu\nu}\dot{\mathcal{G}}_{12}^{\sigma\alpha} - \dot{\mathcal{G}}_{12}^{\mu\alpha}\dot{\mathcal{G}}_{12}^{\sigma\nu}) \\
&+i(\dot{\mathcal{G}}_{22}^{\nu\alpha}\tilde{\mathcal{Z}}^{\mu\sigma} - \dot{\mathcal{G}}_{12}^{\sigma\alpha}\tilde{\mathcal{Z}}^{\mu\nu} + \dot{\mathcal{G}}_{12}^{\sigma\nu}\tilde{\mathcal{Z}}^{\mu\alpha} + \dot{\mathcal{G}}_{12}^{\mu\alpha}\tilde{\mathcal{Z}}^{\sigma\nu} - \dot{\mathcal{G}}_{12}^{\mu\nu}\tilde{\mathcal{Z}}^{\sigma\alpha})\Bigr]
\end{aligned}
\tag{4.6}
$$

Next we decompose $\mathcal{G}_{ij}$ as

$$\mathcal{G}_{ij} = \mathcal{S}_{ij} + \mathcal{A}_{ij} \tag{4.7}$$

[3]Since $G_F, \mathcal{G}_F$ do not occur for the periodic case we delete the subscript "B" in the following.

where $\mathcal{S}$ ($\mathcal{A}$) are its parts even (odd) in F. We can then delete all terms odd in $\tau_1 - \tau_2$ since they vanish upon integration. After using the identity $F\tilde{F} = -g\mathbb{1}$ and some combining of terms, J_5 finally turns into the following, nicely symmetric expression I_5,

$$
\begin{aligned}
I_5^{\mu\nu}(\tau_1,\tau_2) &= i\Big\{\tilde{\mathcal{Z}}^{\mu\nu}k\mathcal{U}_{12}k + \Big[(\tilde{\mathcal{Z}}k)^\mu(\mathcal{U}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big] \\
&\qquad -(\tilde{\mathcal{Z}}\dot{\mathcal{S}}_{12})^{\mu\nu}k\dot{\mathcal{S}}_{12}k - \Big[(\tilde{\mathcal{Z}}\dot{\mathcal{S}}_{12}k)^\mu(\dot{\mathcal{S}}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big]\Big\} \\
&\quad +\frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\Big\{-\dot{\mathcal{A}}_{12}^{\mu\nu}k\mathcal{U}_{12}k - \Big[(\dot{\mathcal{A}}_{12}k)^\mu(\mathcal{U}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big] \\
&\qquad +(\dot{\mathcal{A}}_{22}\dot{\mathcal{S}}_{12})^{\mu\nu}k\dot{\mathcal{S}}_{12}k + \Big[(\dot{\mathcal{A}}_{22}\dot{\mathcal{S}}_{12}k)^\mu(\dot{\mathcal{S}}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big]\Big\}
\end{aligned}
\tag{4.8}
$$

Here in addition to $\mathcal{A}$ and $\mathcal{S}$ we have introduced the combination $\mathcal{U}$,

$$
\mathcal{U}_{12} = \dot{\mathcal{S}}_{12}^2 - (\dot{\mathcal{A}}_{12} - \dot{\mathcal{A}}_{22})(\dot{\mathcal{A}}_{12} + \frac{i}{\mathcal{Z}}) = \frac{1-\cos(\mathcal{Z}\dot{G}_{12})\cos(\mathcal{Z})}{\sin^2(\mathcal{Z})}
\tag{4.9}
$$

Defining also

$$
\hat{\mathcal{A}} \equiv \dot{\mathcal{A}} + \frac{i}{\mathcal{Z}}
\tag{4.10}
$$

this expression can be further compressed to

$$
\begin{aligned}
I_5^{\mu\nu}(\tau_1,\tau_2) &= \frac{\mathcal{Z}\cdot\tilde{\mathcal{Z}}}{4}\Big\{-\hat{\mathcal{A}}_{12}^{\mu\nu}k\mathcal{U}_{12}k - \Big[(\hat{\mathcal{A}}_{12}k)^\mu(\mathcal{U}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big] \\
&\qquad +(\hat{\mathcal{A}}_{22}\dot{\mathcal{S}}_{12})^{\mu\nu}k\dot{\mathcal{S}}_{12}k + \Big[(\hat{\mathcal{A}}_{22}\dot{\mathcal{S}}_{12}k)^\mu(\dot{\mathcal{S}}_{12}k)^\nu + (\mu\leftrightarrow\nu)\Big]\Big\}
\end{aligned}
\tag{4.11}
$$

We can now use the matrix decompositions of $\mathcal{S}, \dot{\mathcal{S}}, \dot{\mathcal{A}}$, given in eq.(3.29) of part I, to write the integrand in explicit form. In this we have a choice between the matrix bases $\{\hat{\mathcal{Z}}_\pm, \hat{\mathcal{Z}}_\pm^2\}$ or $\{\mathbb{1}, F, \tilde{F}, F^2\}$. We will use the former one here since it leads to a somewhat more succinct expression. After the usual rescaling to the unit circle, a transformation of variables $v = \dot{G}_{12}$, and continuation to Minkowski space [4] , we obtain our final result for the vector – axialvector amplitude in a constant field [5] ,

$$
\begin{aligned}
\Pi_5^{\mu\nu}(k) &= \frac{e^3 e_5}{8\pi^2}\mathcal{G}\int_0^\infty ds\, s\, \mathrm{e}^{-ism^2}\int_{-1}^1 \frac{dv}{2}\exp\Bigl[-i\frac{s}{2}\sum_{\alpha=+,-}\frac{\hat{A}^\alpha_{B12}-\hat{A}^\alpha_{B11}}{z_\alpha}\,k\cdot\hat{\mathcal{Z}}^2_\alpha\cdot k\Bigr] \\
&\quad\times\sum_{\alpha,\beta=+,-}\Bigl[\hat{A}^\alpha_{12}\Bigl((\hat{A}^\beta_{12}-\hat{A}^\beta_{22})\hat{A}^\beta_{12}-(S^\beta_{12})^2\Bigr)+\hat{A}^\alpha_{22}S^\alpha_{12}S^\beta_{12}\Bigr] \\
&\qquad\times\Bigl[\hat{\mathcal{Z}}^{\mu\nu}_\alpha k\hat{\mathcal{Z}}^2_\beta k+(\hat{\mathcal{Z}}_\alpha k)^\mu(\hat{\mathcal{Z}}^2_\beta k)^\nu+(\hat{\mathcal{Z}}_\alpha k)^\nu(\hat{\mathcal{Z}}^2_\beta k)^\mu\Bigr]
\end{aligned}
\tag{4.12}
$$

where

$$
\begin{aligned}
S^\pm_{12} &= \frac{\sinh(z_\pm\dot{G}_{B12})}{\sinh(z_\pm)} \\
\hat{A}^\pm_{12} &= \frac{\cosh(z_\pm\dot{G}_{B12})}{\sinh(z_\pm)}, \qquad \hat{A}^\pm_{ii} = \coth(z_\pm) \\
z_+ &= iesa, \qquad a = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}+\mathcal{F}} \\
z_- &= -esb, \qquad b = \sqrt{\sqrt{\mathcal{F}^2+\mathcal{G}^2}-\mathcal{F}} \\
\hat{\mathcal{Z}}_+ &= \frac{aF-b\tilde{F}}{a^2+b^2} \\
\hat{\mathcal{Z}}_- &= -i\frac{bF+a\tilde{F}}{a^2+b^2}
\end{aligned}
\tag{4.13}
$$

As in the vector – vector case, this expression becomes somewhat more transparent if one specializes to the Lorentz system where $\mathbf{E}$ and $\mathbf{B}$ are both pointing along the positive z - axis, $\mathbf{E} = (0,0,E), \mathbf{B} = (0,0,B)$. Here one obtains

[4]For the Maxwell invariants this means $f \to \mathcal{F}$, $g \to i\mathcal{G}$ (we assume $\mathcal{G} \geq 0$).
[5]In undecomposed form this result was already presented in [39].

$$\Pi_5^{\mu\nu}(k) \quad = \quad i\frac{e^2 e_5}{8\pi^2}\int_0^\infty ds \int_{-1}^1 \frac{dv}{2}\,\mathrm{e}^{-is\Phi_0} \sum_{\alpha,\beta=\perp,\parallel} c^{\alpha\beta}\Big[\tilde{F}_\alpha^{\mu\nu}k_\beta^2 + (\tilde{F}_\alpha k)^\mu k_\beta^\nu + (\tilde{F}_\alpha k)^\nu k_\beta^\mu\Big] \tag{4.14}$$

where $z = eBs, z' = eEs,\ k_\perp = (0,k^1,k^2,0),\ k_\parallel = (k^0,0,0,k^3)$,

$$(\tilde{F}_\parallel)^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & B \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -B & 0 & 0 & 0 \end{pmatrix}, \qquad (\tilde{F}_\perp)^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -E & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.15}$$

$$\Phi_0 = m^2 + \frac{k_\perp^2}{2}\frac{\cos(zv) - \cos(z)}{z\sin(z)} - \frac{k_\parallel^2}{2}\frac{\cosh(z'v) - \cosh(z')}{z'\sinh(z')} \tag{4.16}$$

$$\begin{aligned}
c^{\perp\perp} &= z\frac{\cos(zv) - \cos(z)}{\sin^3(z)} \\
c^{\perp\parallel} &= \frac{z\cos(zv)}{\sin(z)}\frac{\cosh(z')\cosh(z'v) - 1}{\sinh^2(z')} - \frac{z\cos(z)\sin(zv)}{\sin^2(z)}\frac{\sinh(z'v)}{\sinh(z')} \\
c^{\parallel\perp} &= -\frac{z'\cosh(z'v)}{\sinh(z')}\frac{\cos(zv)\cos(z) - 1}{\sin^2(z)} - \frac{z'\cosh(z')\sinh(z'v)}{\sinh^2(z')}\frac{\sin(zv)}{\sin(z)} \\
c^{\parallel\parallel} &= -z'\frac{\cosh(z'v) - \cosh(z')}{\sinh^3(z')}
\end{aligned} \tag{4.17}$$

This result can still be slightly simplified using the relations

$$\tilde{F}_\alpha^{\mu\nu} k_\alpha^2 = (\tilde{F}_\alpha k)^\mu k_\alpha^\nu - (\tilde{F}_\alpha k)^\nu k_\alpha^\mu \tag{4.18}$$

($\alpha = \perp, \|$).

By taking the limit $z' \to 0$ in (4.14) we reproduce the known result for the vector – axialvector vacuum polarisation tensor in a constant magnetic field,

$$\begin{aligned}\Pi_5^{\mu\nu}(k) &= \frac{e^2 e_5}{16\pi^2 m^2}\Bigl\{C_\| \Bigl[\tilde{F}^{\mu\nu} k_\|^2 + (\tilde{F}k)^\mu k_\|^\nu + (\tilde{F}k)^\nu k_\|^\mu\Bigr] \\ &\quad + C_\perp \Bigl[\tilde{F}^{\mu\nu} k_\perp^2 + (\tilde{F}k)^\mu k_\perp^\nu + (\tilde{F}k)^\nu k_\perp^\mu\Bigr]\Bigr\}\end{aligned} \tag{4.19}$$

where [6]

$$C_\| = i m^2 \int_0^\infty ds \int_{-1}^1 dv \, \mathrm{e}^{-is\phi_0} \frac{1}{2}(1 - v^2) \tag{4.20}$$

$$C_\perp = i m^2 \int_0^\infty ds \int_{-1}^1 dv \, \mathrm{e}^{-is\phi_0} R \tag{4.21}$$

$$\phi_0 = m^2 + \frac{1 - v^2}{4} k_\|^2 + \frac{\cos(vz) - \cos(z)}{2z\sin(z)} k_\perp^2 \tag{4.22}$$

$$R = \frac{1 - v\sin(z)\sin(vz) - \cos(z)\cos(vz)}{\sin^2(z)} \tag{4.23}$$

The magnetic special case was first obtained in [29] and more recently recalculated in [22]. Our eq. (4.19) can be easily identified with the results given by those authors making use of (4.18).

In a "crossed field", defined by $\mathbf{E} \perp \mathbf{B}, E = B$, both invariants vanish. This case is of importance since a general constant field can be well-approximated by a crossed field at sufficiently high energies (see, e.g., [37]). In a crossed field $F^3 = 0$, so that the power series (3.7) break off after their quadratic terms (see the appendix of part I). The final parameter integral therefore becomes much simpler,

[6] Our definition of $C_\|$ differs by a factor of 2 from the one used in [22].

$$
\begin{aligned}
\Pi_5^{\mu\nu}(k) &= i\frac{e^2 e_5}{4\pi^2}\int_0^\infty ds\int_0^1 du_1 \mathrm{e}^{-i\Bigl[sm^2+sG_{12}k^2-\frac{s^3}{3}G_{12}^2e^2kF^2k\Bigr]}\\
&\times\Bigl\{G_{12}\Bigl[\tilde F^{\mu\nu}k^2+(\tilde Fk)^\mu k^\nu+(\tilde Fk)^\nu k^\mu\Bigr]\\
&-s^2e^2G_{12}^2\Bigl[\tilde F^{\mu\nu}kF^2k+(\tilde Fk)^\mu(F^2k)^\nu+(\tilde Fk)^\nu(F^2k)^\mu\Bigr]\Bigr\}
\end{aligned}
\tag{4.24}
$$

($G_{12}=u_1(1-u_1)$).

Note that our final expressions are manifestly transversal in the vector index. In a field theory calculation this would not be automatically the case, since this amplitude contains the ABJ anomalous triangle graphs [40, 41]. As was already shown in [36] in the present formalism the anomalous divergence is unambiguously fixed to be at the axialvector.

5 Discussion

We have explained in detail how the string-inspired technique can be applied to the computation of one-loop amplitudes involving any numbers of vectors and axialvectors, as well as a constant electromagnetic field. Our explicit calculation of the vector – axialvector two-point amplitude displayed already some of the usual features of the string-inspired technique. In the same way as for the vector – vector case, the Bern-Kosower type partial integration procedure has led to a manifestly gauge invariance result automatically, without the need of performing any subtractions. However, in contrast to the vector – vector case we have not been able to avoid here the explicit evaluation of the Grassmann path integral. In the V-V case this was possible through the application of the "cycle replacement rule", for which no generalization has been found yet to the case of mixed vector – axialvector amplitudes.

Even in the absence of such a generalization it would be of interest to extend the general analysis of the partial integration procedure, performed for the pure vector case in [42], to the mixed vector – axialvector case. Another useful generalization of our results would be the extension to the finite temperature case along the lines of [13, 43, 44].

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Note added: The recent e-print [45] contains a field theory calculation of the vector-axialvector amplitude in a constant field. The final result (13) of this calculation agrees with our (4.14).

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